## Proportional-Integral-Derivative Gain-Scheduling Control of a Magnetic Levitation System

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**Abstract:** The paper presents a gain-scheduling control design procedure for classical Proportional-Integral-Derivative controllers (PID-GS-C) for positioning system. The method is applied to a Magnetic Levitation System with Two Electromagnets (MLS2EM) laboratory equipment, which allows several experimental verifications of the proposed solution. The nonlinear model of MLS2EM is linearized at seven operating points. A state feedback control structure is first designed to stabilize the process. PID control and PID-GS-C structures are next designed to ensure zero steady-state control error and bumpless switching between PID controllers for the linearized models. Real-time experimental results are presented for validation.

**Keywords:** gain-scheduling, magnetic levitation system, Proportional-Integral-Derivative control; real-time experiments.

# 1 Introduction

During the last years several classical and adaptive control structures have been proposed for positioning control system and applications for magnetic levitation systems. Some of these structures are presented in [49], [52], [1], [35], [47], [56], [11], [27], [23], [19], [55], [5]. For example, a high gain adaptive output feedback controller is designed in [5] by introducing two different virtual filters and using back-stepping. Another adaptive control scheme that copes with the modifications of the structural parameters of magnetic levitation systems is suggested in [1].

Gain-scheduling control solutions are popular nowadays, and they are briefly analyzed as follows: fuzzy-based gain scheduling of exact feed-forward linearization control and sliding mode controllers for magnetic ball levitation system are proposed in [28]. A high gain adaptive output feedback control to a magnetic levitation system is discussed in [29]. A Proportional-Integral-Derivative (PID) gain-scheduling controller for second order linear parameter varying, which exclude time varying delay using a Smith predictor is given in [44]. Other interesting adaptive gain scheduling control techniques for real practical applications are given in [6], [54], [7], [17], [53].

The paper is dealing with the position control of a ferromagnetic sphere in a Magnetic Levitation System with Two Electromagnets (MLS2EM) laboratory equipment. A state feedback control structure (SFCS) is first designed in order to stabilize the system by applying the control signal only to the top electromagnet [8]. The simulated external disturbance can be applied to the bottom electromagnet. Starting with [9] the SFCS is next controlled by a Proportional-Integral-Derivative Gain-Scheduling Control (PID-GS-C) structure. In the presented scheme, the proportional, derivative and integral gains are adapted to the modifications of the operating points. The paper proposes relatively simple classical and adaptive control structures which belongs to the general case of linear and nonlinear control system structures [43], [38], [50], [20], [42], [51], [32], [18], [36].

The paper is organized as follows: the nonlinear model and the linearized mathematical model (MM) of MLS2EM are given in Section 2. The proposed control structure is next developed in Section 3. The real-time experimental results are presented in Section 4 and the conclusions are highlighted in Section 5.

### 2 Process modeling

The controlled MLS2EM laboratory equipment includes: two electromagnets (EM1 - the top electromagnet and EM2 - the bottom electromagnet), the ferromagnetic sphere, sensors to detect position of the sphere, computer interface, drivers, power supply unit, connection cables and an acquisition board. The nonlinear state-space MM of ML2SEM is [24]:

$$\begin{split} \dot{p}(t) &= v(t), \\ \dot{v}(t) &= -\frac{i_{EM1}^2(t) \cdot F_{emP1} \cdot \exp(-p(t)/F_{emP2})}{m \cdot F_{emP2}} + g + \frac{i_{EM2}^2(t) \cdot F_{emP1} \cdot \exp(-(x_d - p(t))/F_{emP2})}{m \cdot F_{emP2}}, \\ \dot{i}_{EM1}(t) &= \frac{k_i \cdot u_{EM1}(t) + c_i - i_{EM1}(t)}{\frac{f_{iP1}}{f_{iP2}} \cdot \exp(-p(t)/f_{iP2})}, \\ \dot{i}_{EM2}(t) &= \frac{k_i \cdot u_{EM2}(t) + c_i - i_{EM2}(t)}{\frac{f_{iP1}}{f_{iP2}} \cdot \exp(-(x_d - p(t))/f_{iP2})}, \\ y(t) &= k_m \cdot p(t), \end{split}$$
(1)

where:  $p \in [0, 0.0016]$  - the sphere position (m),  $v \in \Re$ - the sphere speed (m/s),  $i_{EM1}, i_{EM2} \in [0.03884, 2.38]$  - the currents in the top electromagnet (EM1) and bottom electromagnet (EM2) (A),  $u_{EM1}, u_{EM2} \in [0.005, 1]$  - the signals applied to EM1 and EM2, respectively (V), and y - the process output (m), i.e., the measured sphere position. The parameters of the process are determined analytically and experimentally [24], [16].

The model (1) is linearized around seven operating points (o.p.s) with the coordinates  $P^{(j)}(p^{(j)}, v^{(j)}, i_{EM1}^{(j)}, i_{EM1}^{(j)})$  where j is the index of the current operating point, j = 1...7. The linearized state-space models and their matrices are:

$$\begin{split} \Delta \dot{\mathbf{x}}^{(j)} &= \mathbf{A}^{(j)} \Delta \mathbf{x}^{(j)} + \mathbf{b}^{(j)} \Delta \mathbf{u}^{(j)}, \\ \Delta y^{(j)} &= \mathbf{c}^{T(j)} \Delta \mathbf{x}^{(j)}, \\ \Delta \mathbf{x}^{(j)} &= \begin{bmatrix} \Delta p^{(j)} \ \Delta v^{(j)} \ \Delta i^{(j)}_{EM1} \ \Delta i^{(j)}_{EM2} \end{bmatrix}^{T}, \quad \Delta \mathbf{u}^{(j)} &= \begin{bmatrix} \Delta u^{(j)}_{EM1} \ \Delta u^{(j)}_{EM2} \end{bmatrix}^{T}, \\ \mathbf{A}^{(j)} &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ a^{(j)}_{21} & 0 & a^{(j)}_{23} & a^{(j)}_{24} \\ a^{(j)}_{31} & 0 & a^{(j)}_{33} & 0 \\ a^{(j)}_{41} & 0 & 0 & a^{(j)}_{44} \end{bmatrix}, \\ \mathbf{b}^{(j)} &= \begin{bmatrix} \mathbf{b}^{(j)}_{u_{EM1}} \ \mathbf{b}^{(j)}_{u_{EM2}} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ b^{(j)}_{31} & 0 \\ 0 & b^{(j)}_{42} \end{bmatrix}, \\ \mathbf{c}^{T(j)} &= \begin{bmatrix} 1 \ 0 \ 0 \ 0 \end{bmatrix}, \end{split}$$

where T stands for matrix transposition, with the following elements of the matrices  $\mathbf{A}^{(j)}$ and  $\mathbf{b}^{(j)}$ , which depend on  $P^{(j)}$ :

$$\begin{aligned} a_{21}^{(j)} &= \frac{i_{EM1}^{(j)}}{m} \frac{F_{emP1}}{F_{emP2}^2} \exp(-p^{(j)}/F_{emP2}) + \frac{i_{EM2}^{(j)}}{m} \frac{F_{emP1}}{F_{emP2}^2} \exp(-(x_d - p^{(j)})/F_{emP2}), \\ a_{23}^{(j)} &= -\frac{2i_{EM1}^{(j)}}{m} \frac{F_{emP1}}{F_{emP2}} \exp(-p^{(j)}/F_{emP2}), \\ a_{24}^{(j)} &= \frac{2i_{EM2}^{(j)}}{m} \frac{F_{emP1}}{F_{emP2}} \exp(-(x_d - p^{(j)})/F_{emP2}), \\ a_{31}^{(j)} &= -(k_i u_{EM1}^{(j)} + c_i - i_{EM1}^{(j)}) \frac{p^{(j)}}{f_{iP1}} \exp(p^{(j)}/f_{iP2}), \\ a_{31}^{(j)} &= -(k_i u_{EM2}^{(j)} + c_i - i_{EM2}^{(j)}) \frac{p^{(j)}}{f_{iP1}} \exp(p^{(j)}/f_{iP2}), \\ a_{41}^{(j)} &= -(k_i u_{EM2}^{(j)} + c_i - i_{EM2}^{(j)}) \frac{p^{(j)}}{f_{iP1}} \exp((x_d - p^{(j)})/f_{iP2}), \\ a_{31}^{(j)} &= k_i \frac{f_{iP2}}{f_{iP1}} \exp(p^{(j)}/f_{iP2}), \\ b_{31}^{(j)} &= k_i \frac{f_{iP2}}{f_{iP1}} \exp(p^{(j)}/f_{iP2}), \\ b_{42}^{(j)} &= k_i \frac{f_{iP2}}{f_{iP1}} \exp(p^{(j)}/f_{iP2}). \end{aligned}$$

The operating points were chosen as follows such that to belong to the steady-state zone of the sphere position sensor input-output map [33], to cover the usual operating regimes and to avoid the extremities of the input-output map due to instability that may occur:

$$P^{(1)}(0.0063, 0, 1.22, 0.39), P^{(2)}(0.007, 0, 1.145, 0.39), P^{(3)}(0.0077, 0, 1.07, 0.39),$$

$$P^{(4)}(0.0084, 0, 1, 0.39), P^{(5)}(0.009, 0, 0.9345, 0.39), P^{(6)}(0.0098, 0, 0.89, 0.39),$$

$$P^{(7)}(0.0105, 0, 0.83, 0.39).$$
(4)

The transfer function (t.f) of the state-space linearized MM (2) used in the control system design is:

$$H_{PC}^{(j)}(s) = \mathbf{c}^{T(j)}(s\mathbf{I} - \mathbf{A}^{(j)})^{-1}\mathbf{b}_{u_{EM1}}^{(j)} = \frac{k^{(j)}}{\prod\limits_{k=1}^{3} (s - p_k^{(j)})} = \frac{k_p^{(j)}}{\prod\limits_{k=1}^{3} (1 + T_k^{(j)}s)},$$
(5)

where  $k_p^{(j)} = k^{(j)} / \prod_{k=1}^3 p_k^{(j)}$ , **I** is the third-order identity matrix and the time constants are  $T_k^{(j)} = -1/k_p^{(j)}$ , k = 1...3, j = 1...7. The expressions of the t.f.s  $H_{PC}^{(j)}(s)$  are given in [22], [23].

# 3 Control solutions design

In order to support the development of next control solutions, the SFCS is designed for the reduced third-order linear mathematical model ( $u_2 = u_{EM2} = 0$ ) of unstable magnetic levitation system [22]. The first three state variables are kept and they lead to the state vector  $\mathbf{x} = [p \ v \ i_{EM1}]^T$ .

The pole placement method is applied to compute the state feedback gain matrix,  $\mathbf{k}_{c}^{T^{(j)}} = [k_{c1}^{(j)} \ k_{c2}^{(j)} \ k_{c3}^{(j)}]^T$ , j = 1...7. Therefore, for each linearized MM, with the t.f.  $H_{PC}^{(j)}(s)$  (5), the closed-loop system poles  $p_k^{*(j)}$ , k = 1...3, j = 1...7, [22], [23], have been imposed in order to guarantee the stability of the linearized system. With the obtained state feedback gain matrix,  $\mathbf{k}_{cbest}^T = \mathbf{k}_{c_{-5}}^T = [$  66.63 1.62 -0.15 ], two types of closed-loop t.f.s of the new state feedback control structure (nSFCS),  $H_{SFCS_{-5}}^{(j)}(s)$  result as:

$$H_{SFCS_{5}(s)}^{(j)}(s) = H_{x_{5}(s)}^{(j)}(s) = \mathbf{c}^{T(j)}(s\mathbf{I} - \mathbf{A}_{x_{5}(s)}^{(j)})^{-1}\mathbf{b}_{u_{EM1}}^{(j)} = \mathbf{c}^{T(j)}[s\mathbf{I} - (\mathbf{A}^{(j)} - \mathbf{b}_{u_{EM1}}^{(j)}\mathbf{k}_{c_{5}}^{T}k_{AS})]^{-1}\mathbf{b}_{u_{EM1}}^{(j)}$$

$$= \begin{cases} \frac{k_{SFCS_{5}}^{(j)}}{(1+T_{1x_{5}(s)}^{(j)})(1+2\zeta_{5}^{(j)}T_{2x_{5}(s)}^{(j)}+T_{2x_{5}(s)}^{(j)}}s)}, \ j = 1...3, \ j \in \{6,7\}, \\ \frac{k_{SFCS_{5}}^{(j)}}{(1+T_{1x_{5}(s)}^{(j)})(1+T_{2x_{5}(s)}^{(j)})(1+T_{3x_{5}(s)}^{(j)}})}, \ j \in \{4,5\}. \end{cases}$$

$$(6)$$

		nSFCS parameters						
O.p.	$p_{1\_5}^{*(j)}$	$p_{2\_5}^{*(j)}$	$p_{3\_5}^{*(j)}$	$k_{SFCS\_5}^{(j)}$	$T_{1x_{5}}^{(j)}$	$T_{2x_{5}}^{(j)}$	$\zeta_5^{(j)}$	$T_{etax\_5}^{(j)}$
(1)	-0.79 + 1.02i	-0.79-1.02i	-0.10	0.084	0.0988	-	0.6	0.0077
(2)	-0.92 + 0.88i	-0.92-0.88i	-0.13	0.065	0.0778	-	0.7	0.0078
(3)	-1.07 + 0.62i	-1.07-0.62i	-0.16	0.054	0.0618	-	0.9	0.0081
(4)	-1.65	-0.82	-0.21	0.046	0.0485	0.0123	-	-
(5)	-2.32	-0.41	-0.32	0.041	0.0314	0.0244	-	-
(6)	-3.07	-0.28 + 0.16i	-0.28-0.16i	0.038	0.0033	-	0.9	0.0308
(7)	-3.78	-0.23 + 0.20i	-0.23-0.20i	0.034	0.0026	-	0.7	0.0332

Table 1: nSFCS poles and parameters

The new SFCS poles obtained with the state feedback gain matrix  $\mathbf{k}_{c_{5}}^{T}$  and the nSFCS parameters used next in the PID-GS-C structure are given in Table 1.

Due to the fact that the natural SFCS does not contain an I component, so it cannot ensure the zero steady-state control error, the SFCS, as controlled plant, is included in a cascade control structure (CCS) with PID controller in the outer loop. Depending on the operating points, seven control solutions with PID controllers have been designed using pole-zero cancellation. The t.f.s of the designed PID controllers extended with a first order lag filter can be expressed as:

$$H_{PID_{5}(s)}^{(j)}(s) = \begin{cases} \frac{k_{c_{5}(1+2\zeta_{c_{5}}^{(j)}T_{c_{5}}^{(j)}s+T_{c_{5}}^{(j)}s^{2}s^{2})}{s(1+T_{fd_{5}}^{(j)}s)}, j \in \{1, 2, 3, 6, 7\},\\ \frac{k_{c_{5}(1+T_{c1_{5}}^{(j)}s)(1+T_{c2_{5}}^{(j)}s)}{s(1+T_{c1_{5}}^{(j)}s)}, j \in \{4, 5\},\\ \frac{k_{c_{5}(1+T_{c1_{5}}^{(j)}s)}}{s(1+T_{fd_{5}}^{(j)}s)}, j \in \{4, 5\}, \end{cases}$$
(7)

with the tuning parameters  $k_{c_{-5}}^{(j)}$ ,  $T_{c_{-5}}^{(j)}$ ,  $T_{c1_{-5}}^{(j)}$ ,  $T_{c2_{-5}}^{(j)}$  and  $T_{fd_{-5}}^{(j)}$ :

$$k_{c_{-5}}^{(j)} = \begin{cases} 1/(2 \cdot k_{SFCS_{-5}}^{(j)} \cdot T_{etax_{-5}}^{(j)}), \quad j = 1...3 \\ 0.05/(2 \cdot k_{SFCS_{-5}}^{(j)} \cdot T_{\Sigma x_{-5}}^{(j)}), \quad j \in \{4,5\} \\ 0.01/(2 \cdot k_{SFCS_{-5}}^{(j)} \cdot T_{1x_{-5}}^{(j)}), \quad j \in \{6,7\}, \end{cases}$$

$$\begin{cases} T_{c_{-5}}^{(j)} = T_{etax_{-5}}^{(j)}, \quad j \in \{1,2,3,6,7\}, \\ T_{c_{1-5}}^{(j)} = T_{1x_{-5}}^{(j)}, \quad j \in \{4,5\}, \\ T_{c_{2-5}}^{(j)} = T_{2x_{-5}}^{(j)}, \quad j \in \{4,5\}, \\ T_{fd_{-5}}^{(j)} = 0.1 \cdot T_{1x_{-5}}^{(j)}, \quad j = 1...7, \end{cases}$$

$$(8)$$

Due to the oscillatory regime, the t.f.s coefficients  $k_{c-5}^{(j)}$  must be adjusted. The numerical values of PID-C parameters and the system performance indices – overshoot and settling time – are synthesized in Table 2.

Due to the process nonlinearities, which also depend on the operating point, the switching from one PID controller to another one is a useful solution to ensure improved performance according to [48]. Therefore, a PID-GS-C structure illustrated in Figure 1 is designed. The PID-GS-C structure is developed on the basis of the PID controller (with the notation PID-C in Figure 1), with the t.f.:

$$\Delta u_{1x}(t) = k_p(t)e(t) + k_i(t) \int e(t)dt + k_d(t)\dot{e}(t),$$
(9)

where  $k_p$  is the proportional gain,  $k_d$  is the derivative gain and  $k_i$  is the integral gain.

O.p.	PID controller tuning		PID control system		
	parameters		performance indices		
	$k_c^{(j)}{}_5$	$T_{c\_5}^{(j)}$	$t_{r}^{(j)}{}_{5}$	$\sigma_{1-5}^{(j)}$	
(1)	60.65	0.009	4,25	0,24	
(2)	98.23	0.008	4,25	$0,\!24$	
(3)	150.83	0.006	$^{4,5}$	$0,\!24$	
(4)	89.88	0.005	$^{5,0}$	$0,\!23$	
(5)	28.67	0.003	$^{4,0}$	$0,\!23$	
(6)	40.98	0.003	$^{3,5}$	$0,\!24$	
(7)	111.24	0.003	$^{4,0}$	$0,\!24$	
{kpma	{ε,β,η, x, kpmin, kdm kimax, kimi	ax, kdmin, ki	PID PID-	-C GS-C	SFCS

Table 2: PID controller parameters

Figure 1: Block diagram of PID-GS-C system structure

As shown in Figure 1, the gain-scheduling (GS) block uses the reference input r(t) and the control error e(t) as input variables, and the tuning parameters of the PID-C, namely  $k_p$ ,  $k_d$  and  $k_i$ , as output variables. The PID tuning parameters are obtained as follows:

$$k_{p}(t) = k_{p \max} - (k_{p \max} - k_{p \min}) \exp[-(\alpha(t)|e(t)|)],$$
  

$$k_{d}(t) = k_{d \max} - (k_{d \max} - k_{d \min}) \exp[-(\alpha(t)|e(t)|)],$$
  

$$k_{i}(t) = (1 - \alpha(t))k_{i \max}.$$
(10)

The parameters  $k_{p \max}$ ,  $k_{p \min}$ ,  $k_{d \max}$ ,  $k_{d \min}$ ,  $k_{i \max}$  and  $k_{i \min}$  are determined from the PID-GS-C tuning parameters:

$$k_{p \max} = \begin{cases} \max[k_{c_{-5}}^{(j)} \cdot (T_{c_{1}}^{(j)} + T_{c_{2}}^{(j)})], j \in \{4, 5\}, \\ \max[2 \cdot k_{c_{-5}}^{(j)} \cdot \zeta_{c_{-5}}^{(j)} \cdot T_{c_{-5}}^{(j)}], j \in \{1, 2, 3, 6, 7\}, \end{cases}$$

$$k_{p \min} = \begin{cases} \min[k_{c_{-5}}^{(j)} \cdot (T_{c_{1}}^{(j)} + T_{c_{2}}^{(j)})], j \in \{4, 5\}, \\ \min[2 \cdot k_{c_{-5}}^{(j)} \cdot \zeta_{c_{-5}}^{(j)} \cdot T_{c_{2}}^{(j)}], j \in \{1, 2, 3, 6, 7\}, \end{cases}$$

$$k_{d \max} = \begin{cases} \max(k_{c_{-5}}^{(j)} \cdot T_{c_{1}}^{(j)} + T_{c_{2}}^{(j)}), j \in \{4, 5\}, \\ \max(k_{c_{-5}}^{(j)} \cdot T_{c_{1}}^{(j)} + T_{c_{2}}^{(j)}), j \in \{4, 5\}, \\ \max(k_{c_{-5}}^{(j)} \cdot T_{c_{1}}^{(j)} + T_{c_{2}}^{(j)}), j \in \{4, 5\}, \\ \min(k_{c_{-5}}^{(j)} \cdot T_{c_{1}}^{(j)} + T_{c_{2}}^{(j)}), j \in \{4, 5\}, \\ \min(k_{c_{-5}}^{(j)} \cdot T_{c_{2}}^{(j)}), j \in \{1, 2, 3, 6, 7\}, \end{cases}$$

$$k_{i \max} = \max(k_{c_{-5}}^{(j)}), k_{i \min} = \min(k_{c_{-5}}^{(j)}), j = 1...7. \end{cases}$$

$$(11)$$

The parameter  $0 \le \alpha(t) \le 1$  is included in order to have a smooth and continuous variation of the switching from one PID controller to another one. The following equation is used to get the value of this parameter:

$$\alpha(t) = \tanh(\eta\beta(t)) = [\exp(2\eta\beta(t)) - 1] / [\exp(2\eta\beta(t)) + 1],$$
(12)

where the parameter  $\eta$  determines the rate at which  $\alpha(t)$  changes between 0 and 1 and chosen in order to ensure a certain dynamics of the variation of  $\alpha(t)$ . The parameter  $\beta(t)$  is set in terms of [35]:

$$\beta(t) = \begin{cases} 1, & |e(t)| > \xi, \\ 0, & |e(t)| < \xi, \end{cases}, \quad \xi = 0.9 \cdot r(t).$$
(13)

As shown in [35], to design the PID-GS-C structure, the following conditions can be taken into account: when the system is in steady-state error (i.e., |e(t)| is large),  $k_{p\max}$  and  $k_{i\min}$  are activated in order to produce a large control signal and to overcome the undesirable oscillation and overshoot; during the steady-state regime,  $k_{i\max}$  and  $k_{p\min}$  are activated to obtain a small value of |e(t)| and to overcome the undesirable problem of overshoot.

### 4 Experimental results

All control structures, namely, SFCS, PID-C and PID-GS-C, were tested on the nonlinear laboratory system and validated by real-time experiments. In all cases, the reference input was set to 0.007 m from the top electromagnet and the control structures responses were tested on the time frame of 20 s. The responses of the sphere position, the current and the control signal in the top electromagnet were plotted.

The values of the parameters of the PID-GS-C block in Figure 1 are  $k_{p \max} = 5516$ ,  $k_{p \min} = 0.6$ ,  $k_{d \max} = 0.1225$ ,  $k_{d \min} = 0.0036$ ,  $k_{i \max} = 151$ ,  $k_{i \min} = 0$  and  $\eta \in \{0.001, 0.1\}$ . They have been obtained using several experiments such that to get the best values in the context of a compromise to tradeoff to overshoot. But other empirical performance indices can considered in this tuning.

The following experimental scenarios were considered and performed:

(a) The state feedback control system structure with the best state feedback gain matrix was tested on the laboratory equipment and the results are presented in Figure 2.

(b) The PID controller designed at seven operating points was tested on laboratory equipment. The experimental results of the control system with PID controller designed only for three o.p.s defined in Tables 1 and 2, i.e., (1), (5) and (7) are presented as responses of several variables measured in the laboratory setup in Figures 3 to 5 as follows: the control signal versus time in Figure 3, he current through EM1 versus time in Figure 4 and the controlled output versus time in Figure 5. These results are better in comparison with the results presented in Figure 2 because of the PID-C, which ensures the zero steady-state control error and the reference input is tracked. The pair of complex conjugated poles from the cases that correspond to the o.p.s (1) to (3) (similarly to the o.p.s (6) and (7)) and the nonlinearities of the process lead to oscillations at the beginning of transient responses and during the real-time experiments.

(c) The experimental results of the control system with PID-GS controller are presented as responses of several variables measured in the laboratory setup in Figures 6 to 8 as follows: the control signal versus time in Figure 6, he current through EM1 versus time in Figure 7 and the controlled output versus time in Figure 8. These results are better in comparison with the results presented in Figures 3 to 5 because of the PID-GS controller that ensures improved dynamic behavior characterized by smaller overshoot and settling time.



Figure 2: Real-time experimental results of state feedback control system structure with the best state feedback gain matrix



Figure 3: Control signal  $u_1 = u_{EM1}^{(j)}$ ,  $j \in \{3, 5, 6\}$ , versus time for control systems with PID controller for o.p.s. (3), (5) and (6)



Figure 4: Current  $i_{EM1}$  versus time for control systems with PID controller designed for o.p.s. (3), (5) and (6)



Figure 5: Sphere position p versus time for control systems with PID controller designed for o.p.s. (3), (5) and (6)



Figure 6: Control signal  $u_1 = u_{EM1}^{(j)}$  versus time for control systems with PID-GS controller



Figure 7: Current  $i_{EM1}$  versus time for control systems with PID-GS controller



Figure 8: Sphere position p versus time for control systems with PID-GS controller

## 5 Conclusions

The paper has presented three control structures developed to control the position of the sphere in an MLS2EM laboratory setup. In order to use relatively simple control structures, the presented nonlinear model of the MLS2EM was linearized around seven operating points. To stabilize the process, a state feedback control structures was designed and the best state feedback gain matrix was found.

Since the SFCS does not ensure the zero steady-state control error, seven PID controllers were designed. To ensure the switching between different PID controllers, a PID-GS controller was developed and implemented. All control system structures were tested on the nonlinear model, accepting the main values of the parameters given in [24]. This paper has considered a low-cost implementation of the controllers but this must be viewed in connection with the complexity of the control algorithms, and several approaches can be used [10], [12], [37], [13], [15], [31], [46].

The real-time experimental results prove that the PID-GS-C structure discussed in this paper guarantees the improvement of control system performance regarding to step modifications of reference input. They ensure zero steady-state control error, small overshoot and settling time. As shown in the previous section, the choice of the parameters of the PID-GS-C block in Figure 1 has been carried out by conducting several experiments such that to aim the best values from the point of view of the compromise to the achievement of best overshoot and settling time. This is a limitation of the control system structure presented in this paper.

The systematic choice of the parameters of PID-GS-C will represent a direction of future research by ensuring the optimal tuning using classical [4], [22], [39], [2], [25], [14] and modern optimization algorithms [40], [41], [45], [26], [33], [30], [34], [21], [3] or their combinations. Future research will also be focused on the design of control systems with PI(D) fuzzy gain-scheduling controllers, Takagi-Sugeno fuzzy controllers and hybrid structures including sliding mode control and gain-scheduling control for improved performance indices.

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