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EXCHANGE-TRADED FUNDS (ETFS) OPTIMIZATION WITH RISK PARITY STRATEGIES

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Abstract: *This paper explores the application of the Risk Parity methodology to portfolios constructed using Exchange-Traded Funds (ETFs). In the world where the market is becoming more and more global, it is becoming more dynamic in terms of portoflio recalibrations, bringing resilience in terms of values. In other words, is it important to consider only a short holding period, and adapt the asset allocationn. In opposition to typical capital-weighted techniques, risk parity reallocates portfolio weights to ensure that each asset contributes an equal amount of risk. The study builds and assesses risk parity portfolios using ten assets that comprise the majority of the EA Bridgeway Blue Chip ETF (BBLU). The process entails assessing the volatilities and correlations of specific ETFs, adjusting portfolio weights to balance risk contributions, and back testing performance under different market scenarios. Empirical data shows that risk parity portfolios outperform standard allocation approaches in terms of diversification, drawdowns, and the stability of risk-adjusted returns. This research illustrates Risk Parity's potential as a strong framework for managing ETF portfolios in both institutional and individual investing settings.*

Keywords: *Risk Parity, ETF, Portfolio Optimization, Asset allocation, Risk.*

Introduction

Modern finance focuses on risk management and portfolio development. Traditional portfolio allocation approaches, such as the 60/40 equity-bond split, prioritize capital allocation, which frequently results in an imbalance in risk contributions across asset classes (Maillard et al., 2010, pp.2). This strategy can lead to an overreliance on more volatile assets, such as stocks, while undervaluing the potential diversification benefits of less volatile asset classes, such as bonds or commodities. Risk Parity (RP) provides an alternative paradigm that reallocates weights to ensure that each asset contributes equally to overall portfolio risk, resulting in a more balanced and diversified portfolio structure (Qian, 2011, pp.2).

The application of Risk Parity to portfolios built with Exchange-Traded Funds (ETFs) is a significant step toward implementing advanced portfolio methods. ETFs are well-known for their cost-effectiveness, liquidity, and accessibility, making them an excellent vehicle for adopting systematic approaches such as Risk Parity. ETFs also give exposure to a diverse variety of asset classes, such as stocks, bonds, commodities, and real estate, which is critical for attaining broad diversification (Hill et al., 2015, pp.6).

This article brings at the practical use of the Risk Parity technique using ETFs.It focus es on creating portfolios that balance risk contributions across specific ETFs while taking into account real world restrictions such as transaction costs and leverage requirements. The Conditional Value at Risk (Rockfellar & Uryasev, 2000), or the Expected Shortfall (Artzner & Delbaen, 1999) and the traditional mean-variance (Markowitz, 1952), will result in a strong concentration on a limited number of assets and poor performance during the out-of-sample period. These models also rely on predicted returns (Markowitz, 1978, pp.2), which are usually calculated using financial models or historical data. These projections are not without uncertainty, though, and can be impacted by a number of variables, including shifts in investor mood, market circumstances, and economic developments. Suboptimal portfolio allocations and performance may result if the predicted returns utilized in the portfolio model prove to be incorrect. According to Merton (1980, pp.2), portfolio models that mainly depend on expected returns may be extremely vulnerable to shifts in those assumptions.

The ten biggest blue chip stock part of EA Bridgeway Blue Chip ETF (BBLU) are selected and optimized in the Risk Parity model using $CVaR_{\alpha}(x)$ as a risk measure. The partial derivatives of the Conditional Value at Risk can be calculated using approximation techniques (Tasche, 2000, pp.3). The comparison using Risk Parity methods is made with several risk measurements (Conditional Value at Risk and standard deviation) in order to obtain a comprehensive pattern. Although the findings are quite similar, it takes a lot less time to calculate Risk Parity with Conditional Value at Risk.

Research methodology

This section outlines the empirical framework for evaluating the six portfolio strategies for the ten biggest blue-chip stocks comprising the EA Bridgeway Blue Chip ETF (BBLU). The goal is to compare their risk-adjusted performance, diversification benefits, and robustness. To provide a complete framework for the portfolio models, the following are considered:

1. 1/N Naïve Portfolio with the same weight for each asset (10% of each asset);
2. Mean variance without the expected return constrain (MV);
3. Minimum CVaR (Andersson et al., 2000) without the expected return constrain;
4. Risk Parity with standard deviation (RP-std);
5. Risk Parity with Expected shortfall or CVaR (RP-CVaR);
6. Worst case Risk Parity CVaR (RP-CVaR Naïve);

The last one is a special case for the worst-case scenario (highest CVaR), useful as an upper bound (Colucci, 2013). A similar study was conducted with cryptocurrencies (Veliu & Aranitasi, 2024).

In all these models, the constraint of expected returns is removed, so the minimum variance MV and minimum CVaR are at the smallest possible value of the risk measure.

For a portfolio with n assets, each weight x_i and $\mathcal{R}(x)$ as a risk measure for the portfolio, the vector of the weights is given by:

$$x = (x_1, x_2, \ldots, x_n).$$

In the literature (Maillard et al., 2010 p.1), the use of Risk Parity is the case with the standard deviation as risk measure. For a portfolio with n assets and weights $x = (x_1, x_2, \ldots, x_n)$, the standard deviation is:

$$\mathcal{R}(x) = \sigma_P(x) = \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} x_i x_j \sigma_{ij}} = \sqrt{x'\Omega x}$$

where Ω is the covariance matrix. The marginal risk contribution of the *i* asset:

$$MRC_i(x) = \frac{\partial \sigma_P(x)}{\partial x_i} = \frac{\partial \sigma_i^2 + \sum_{j=1}^{n} x_i \sigma_{ij}}{\sigma_P(x)} = \frac{(\Omega x)_i}{\sqrt{x'\Omega x}}$$

and the total risk contribution:

$$TRC_i(x) = x_i \frac{\partial \sigma_P(x)}{\partial x_i} = x_i \frac{\partial \sigma_i^2 + \sum_{j=1}^{n} x_i \sigma_{ij}}{\sigma_P(x)} = x_i \frac{(\Omega x)_i}{\sqrt{x'\Omega x}}$$

The following optimization problem can be used to construct the Risk Parity model:

$$x^* = \arg\min \sum_{i=1}^{n}\sum_{j=1}^{n} \left(TRC_i(x) - TRC_j(x)\right)^2$$

$$\sum_{i=1}^{n} x_i = 1$$

$$x \geq 0$$

To guarantee the existence of the partial derivatives of $CVaR_\alpha(x)$ some assumptions are needed on the distribution of the random vector $R = (r_1, r_2, \ldots, r_n)$.

The conditions for quantile of the portfolio return $X = R'x = \sum_{i=1}^{n} x_i r_i$ should be differentiable respect to the weights x_i. These *i*-th asset return r_i given the others is measured as follow:

$$r_{i,t+1} = \frac{P_{i,t+1} - P_{i,t}}{P_{i,t}}$$

From the definition of $CVaR_\alpha(x)$ (Rockfellar & Uryasev, 2000, pp.2) is as follow:

$$CVaR_\alpha(x) = \frac{1}{\alpha}\int_0^\alpha VaR_v(x)\mathrm{d}v$$

Thus, partial derivatives are calculated as follow:

$$\frac{\partial CVaR_\alpha(x)}{\partial x_i} = \frac{1}{\alpha}\int_0^\alpha \frac{\partial CVaR_\alpha(x)}{\partial x_i}\mathrm{d}v = -\frac{1}{\alpha}\int_0^\alpha E[r_i|-R'x = VaR_\alpha(x)]\mathrm{d}v =$$

$$-\frac{1}{\alpha}\int_0^\alpha E[r_i|X = q_\alpha(X)]\mathrm{d}v = -\ E[r_i|X \leq -VaR_\alpha(x)]$$

The Total Risk contribution for each asset *i* of a portfolio is given from the following expression:

$$TRC_i^{CVaR_\alpha(x)}(x) = x_i \frac{\partial CVaR_\alpha(x)}{\partial x_i}$$

The expression in case of continuous returns distribution is the following:

$$TRC_i^{CVaR_\alpha(x)}(x) = -x_i E[r_i|X \leq -VaR_\alpha(x)]$$

$$CVaR_\alpha(x) = \sum_{i=1}^{n} TRC_i^{CVaR_\alpha(x)}(x) = -\sum_{i=1}^{n} x_i E[r_i|X \leq -VaR_\alpha(x)]$$

Numerical approximation for estimating $VaR_\alpha(x)$ and $CVaR_\alpha(x)$ Risk Parity using historical data the following assumption are neccessary.
The calculation of $VaR_\alpha(x)$ and $CVaR_\alpha(x)$ of portfolio returns as follows:

$$VaR_\alpha(x) \approx -r_{\mathrm{p}\lfloor\alpha\mathrm{T}\rfloor}^{\mathrm{sorted}}$$

$$CVaR_\alpha(x) \approx -\frac{1}{\alpha\mathrm{T}}\sum_{\mathrm{j}=1}^{\lfloor\alpha\mathrm{T}\rfloor} r_{\mathrm{p\,j}}^{\mathrm{sorted}}$$

where the *i*-th asset return r_i consist of *T* number outcomes r_{ji} with *i*=1,.....,*n* and *j*=1,...,T.
The vector of the observed portfolio returns is $R_P = (r_{p1}, \ldots\ldots, r_{pT})$ where:
$r_{pj} = x'r^j$ with j=1,....,T where $r^j = (r_{j1}, \ldots\ldots, r_{jT})$.
where α is level of significance and $\mathrm{r}_{\mathrm{p\,j}}^{\mathrm{sorted}}$ are the sorted portfolio returns such as

$$r_{p\,1}^{sorted} \leq r_{p\,2}^{sorted} \leq \cdots r_{p\,j}^{sorted} \leq \cdots \leq r_{p\,j}^{sorted}$$

With the time series observation, the approximation of the partial derivatives $CVaR_\alpha(x)$ for each asst *i* becomes:
$\frac{\partial CVaR_\alpha(x)}{\partial x_i} \approx -\frac{1}{\alpha\mathrm{T}}\sum_{\mathrm{k}=1}^{\lfloor\alpha\mathrm{T}\rfloor} r_{\mathrm{k\,i}}^{\mathrm{sorted}}\ \forall\ i=1,\ldots,n$
and then the total risk contribution of asset *i* is

$$TRC_i^{CVaR_\alpha(x)}(x) = x_i\frac{\partial CVaR_\alpha(x)}{\partial x_i} \approx -\frac{1}{\lfloor\alpha\mathrm{T}\rfloor}x_i\sum_{\mathrm{k}=1}^{\lfloor\alpha\mathrm{T}\rfloor} r_{\mathrm{k\,i}}^{\mathrm{sorted}}$$

where $r_{\mathrm{k\,i}}^{\mathrm{sorted}}$ are the corresponding returns of asset *i* to the sorted portfolio returns.
In the rolling windows, in order to the measure the performance the following calculation $\mu_T^c(R_P)$ is the compounded return over the whole period (terminal compound return) (Bacon, 2008, pp.130).

$$\mu_k^c(R_P) = \prod_{j=1}^{k}\left(1 + r_{pj}\right) - 1$$

To check if the portfolios are well diversified, three diversification measures are considered (Caporin et al., 2012).
If the allocation is as follows, $x = (x_1, x_2, \ldots, x_n)$ with the constraint $\sum_{i=1}^n x_i = 1$ in case short sales not allowed $(x_i \geq 0)$. The diversification measure is the Herfindal index:

$$D_{Her} = 1 - xx'$$

In the same way, the diversification measure is given by (Bera & Park, 2004 p.2).

$$D_{BP} = -\sum_{i=1}^{n} x_i \log(x_i) = \sum_{i=1}^{n} x_i \log(\frac{1}{x_i})$$

The D_{BP} takes value between 0 (fully concentrated in one asset) and log(*n*) for the Naïve portfolio.
Another important aspect is the consideration of the transaction costs, and for that, the estimation of the turnover of the portfolio:

$$TO = \sum_{i=1}^n |x_i^{t+1} - x_i^t|,$$

where x_i^t denotes the weight of asset *i* at time *t*.

Results

In this study, the dataset covers the period from January 9, 2024 to January 8, 2025 with daily frequency. The data is available at nasdaq.com.

Table 1. *Top EA Bridgeway Blue Chip ETF (BBLU)*

1	Meta Platforms, Inc. Class A Common Stock (META)
2	NVIDIA Corporation Common Stock (NVDA)
3	JP Morgan Chase & Co. Common Stock (JPM)
4	Broadcom Inc. Common Stock (AVGO)
5	Tesla, Inc. Common Stock (TSLA)
6	Apple Inc. Common Stock (AAPL)
7	Visa Inc. (V)
8	Microsoft Corporation Common Stock (MSFT)
9	Eli Lilly and Company Common Stock (LLY)
10	Wells Fargo & Company Common Stock (WFC)

Source: nasdaq.com

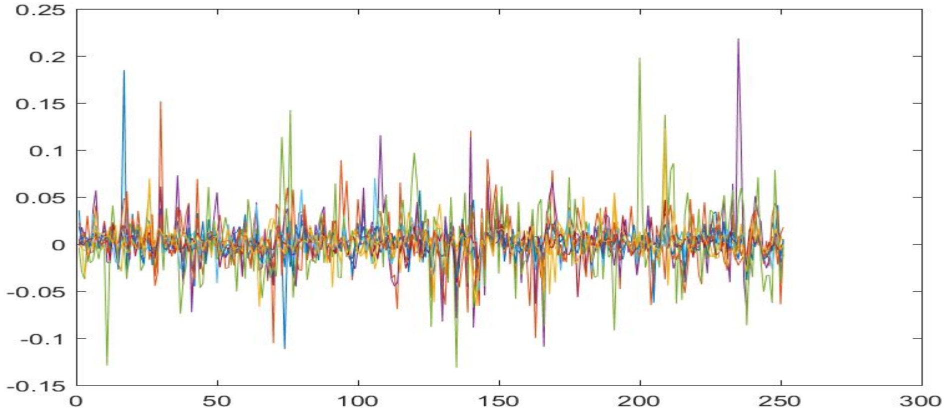
The reason these stocks are selected is because they comprise the top components of the EA Bridgeway Blue Chip ETF (BBLU). In this way, it will be interesting to allocate the optimal weights using different strategies and compare their performance among them.

Figure 1. *The Heatmap of the correlation matrix*

	META	*NVIDIA*	*JPM*	*AVGO*	*TESLA*	*APPLE*	*VISA*	*Micro*	*ELI*	*WFC*
META	100.0%									
NVIDIA	83.9%	100.0%								
JPM	84.7%	88.8%	100.0%							
AVGO	81.8%	82.1%	80.0%	100.0%						
TESLA	68.7%	62.3%	79.3%	81.5%	100.0%					
APPLE	71.7%	83.5%	81.6%	85.7%	78.7%	100.0%				
VISA	67.6%	48.8%	71.0%	60.9%	80.0%	47.5%	100.0%			
Micro	50.1%	61.3%	43.7%	56.2%	36.8%	50.2%	22.0%	100.0%		
ELI	50.3%	66.1%	41.7%	44.1%	4.1%	53.3%	-12.8%	53.2%	100.0%	
WFC	71.9%	73.7%	87.9%	65.7%	73.3%	56.2%	78.8%	39.8%	11.1%	100.0%

Figure 1 presents a heatmap-style correlation matrix where each cell contains a numerical value representing the correlation between two variables. All diagonal values are 100% (in red), which reflects perfect correlation of each variable with itself. Red: Represents the highest positive correlation (close to 100%). Green: Represents weaker or negative correlations. Yellow/Orange: Represents moderate positive correlations. The heatmap suggests a symmetric matrix (correlation matrices are symmetric by definition). The variables are represented both horizontally and vertically, indicating the relationship between the same set of items. Many variables exhibit strong positive correlations (>80%) as indicated by the red and orange cells. Clusters of high correlation suggest that certain variables behave similarly or share strong linear relationships. Yellow cells indicate moderate correlations (40%–70%). These relationships are still positive but weaker than the stronger clusters. Moderate correlations could imply indirect relationships or shared dependence on other factors. The -12.8% correlation stands out as a clear example of an inverse relationship between two variables. This suggests that as one variable increases, the other tends to decrease. Negative correlations, though rare in this matrix, might signal opposing behavior or complementary roles like in the case of ELI with VISA.

Figure 2. *The graph of the returns of the ten assets*



The Figure 2 represents the daily returns of 10 assets plotted over a time period of approximately 251 days. The daily returns fluctuate around zero, with the majority of returns falling within the range of -0.15 to 0.25. The spikes indicate days with significant movements in returns, likely driven by market events. From a visual inspection, there are some similar patterns across the lines, suggesting potential correlation between certain assets.

Before creating a rolling window, in Table 2 are showed the first iteration using 125 observations:

Table 2. *The allocation in % in of each of assets*

Optimization Model	META	NVIDIA	JPM	AVGO	TESLA	APPLE	VISA	Micro	ELI	WFC	TOTAL
R.P. with std	6.17%	5.17%	13.93%	5.27%	5.83%	12.19%	13.36%	11.39%	12.45%	14.24%	1
Minimum Variance	0.00%	0.06%	14.35%	0.02%	0.07%	14.41%	31.43%	10.41%	15.48%	13.78%	1
R.P. with CVaR	7.62%	5.65%	12.04%	6.60%	4.92%	11.63%	16.18%	11.68%	10.69%	12.98%	1
Minimum CVaR	0.65%	4.44%	3.41%	0.00%	0.00%	15.25%	27.27%	9.06%	14.71%	25.20%	1
Naïve	10%	10%	10%	10%	10%	10%	10%	10%	10%	10%	1
R.P. CVaR naïve	7.94%	4.19%	13.70%	4.29%	6.06%	13.29%	13.75%	10.49%	13.49%	12.81%	1

Table 2 represents the portfolio weights for 10 assets under different optimization models. The Risk Parity (R.P. with std) with Standard Deviation, which aims to balance risk contribution across assets using standard deviation as the risk metric, is Heavily weighted on JPM (13.93%), Apple (13.36%), and ELI (12.45%). Minimum Variance focuses heavily on Apple (31.43%) and JPM (14.35%), with near-zero weights for META and others. Risk Parity with CVaR (Conditional Value-at-Risk) is similar to R.P. but uses CVaR to account for tail risk. JPM (12.04%) and VISA (16.18%) have higher weights, reflecting their perceived stability under extreme conditions.

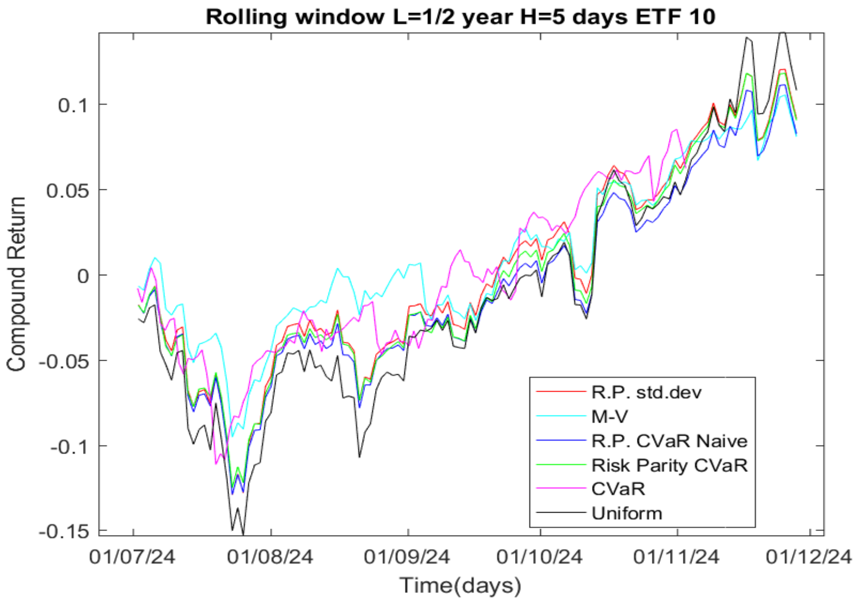
Minimum CVaR minimizes portfolio tail risk. VISA (27.27%) and WFC (25.20%) dominate the allocation, likely reflecting their low downside risk. Naïve (Equal Allocation) assigns an equal 10% weight to all assets, disregarding risk or return considerations.

R.P. CVaR Naïve is a special case in which has the worst-case scenario (highest CVaR, useful as an upper bound (Colucci, 2013). JPM (13.7%) and VISA (13.75%) maintain higher weights but are more evenly distributed compared to other R.P. models.

Naïve allocation provides the most diversified portfolio, while others heavily concentrate on specific assets. Minimum Variance and Minimum CVaR models allocate significant weights to assets with perceived lower risk. The JPM & VISA Dominance consistently receives higher weights in most models, indicating their importance in minimizing risk or balancing contributions. Tesla and AVGO tend to receive lower weights, likely due to higher volatility or correlation with other assets.

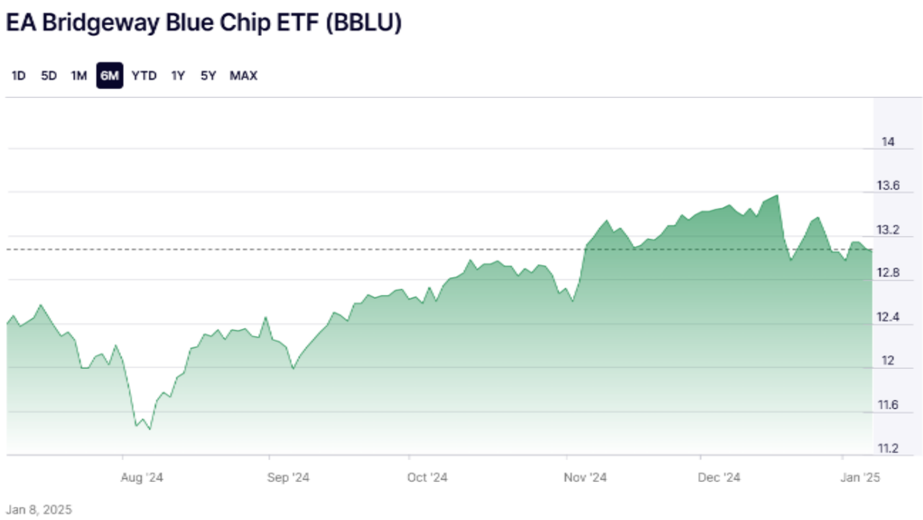
In order to keep the portfolio updated, the calculation is made each week, based on the past 25 weeks, the asset allocation. The rolling window is made with L=125 days (6 months) and H=5 days, where L are the daily observations used to estimate the weights and H is the holding period for the performance of the portfolio using the compound return. The performance is given by the following graph 1.

Graph 1. *The performance of the portfolios in ETF fund using the compound return.*



Source: Computed with Matlab

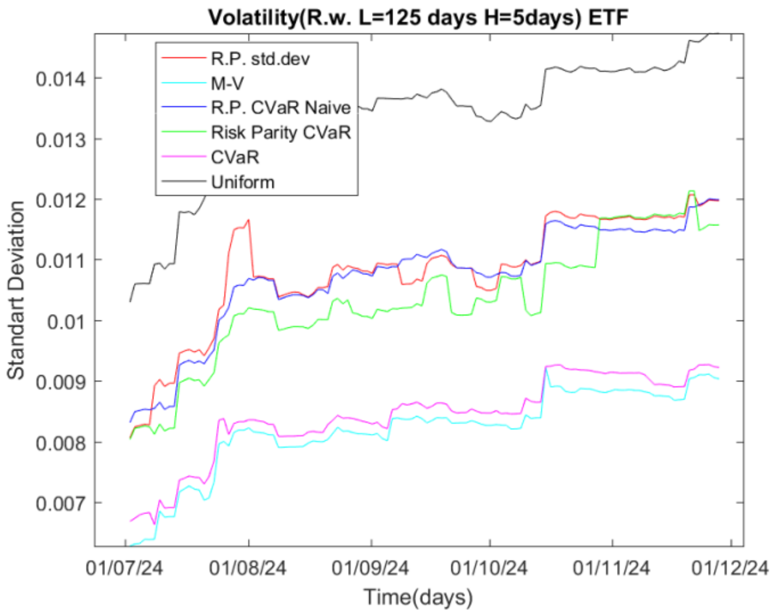
Graph 2. *The performance of EA Bridgeway Blue Chip ETF (BBLU)*



Source: Nasdaq.com

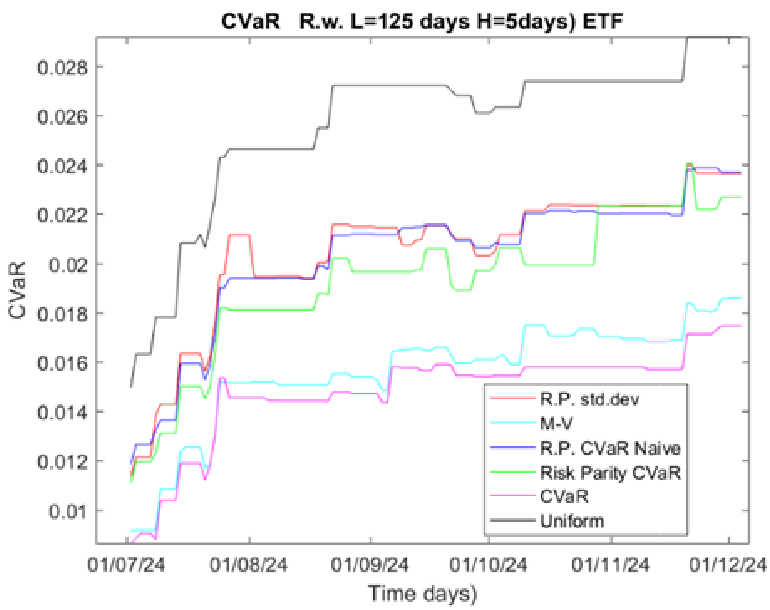
Graph 1 aims to evaluate how different portfolio optimization strategies perform over a rolling half-year period, with a five-day investment horizon, highlighting the trade-offs between return and risk for each strategy. Graph 1 presents a detailed comparison of the performance of different portfolio optimization strategies over time. Most strategies show similar patterns with noticeable ups and downs, reflecting market dynamics, for which are represented using the graph 2. By the end of the time period, some strategies (e.g., "Risk Parity CVaR" and "M-V") outperform others, reaching higher compound returns. The "Uniform" strategy (black) generally underperforms compared to other approaches. The first graph uses compound returns as the performance metric, whereas the second graph uses the ETF price.

Graph 3. *The riskiness of the portfolios measured by the standard deviation (Volatility)*



Source: Computed with Matlab

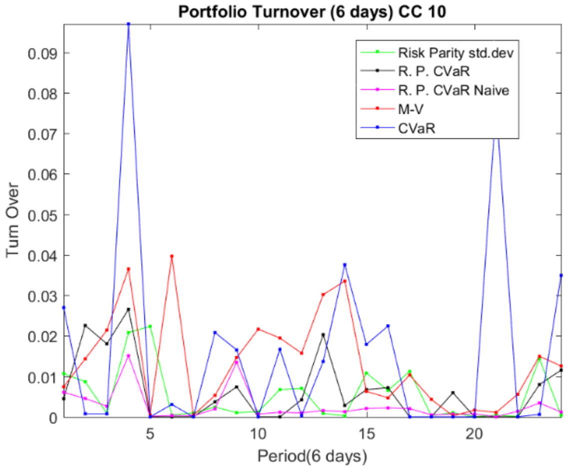
Graph 4. *The riskiness of the portfolios measured by the CVaR*



Source: Computed with Matlab

The CVaR graph focuses on extreme downside risks, while the volatility graph examines total risk. If a strategy shows lower volatility (graph 3), it may also show lower CVaR (graph 4), but this is not always guaranteed. For instance, strategies designed to reduce overall risk (e.g., Risk Parity) might perform differently in tail-risk scenarios. The CVaR values are higher than volatility values, consistent with CVaR capturing extreme losses beyond a certain threshold, whereas volatility reflects average dispersion. Comparing the two graphs can reveal how a strategy balances overall risk (volatility) with tail risk (CVaR). A strategy with low CVaR but higher volatility may be suitable for risk-averse investors, while others might prefere strategies with low volatility for consistent returns. The patterns in the two graphs are likely similar in overall behavior, as both measure aspects of risk across time and are influenced by the same portfolio strategies. However, there may be some nuanced differences depending on how each metric reacts to market changes.

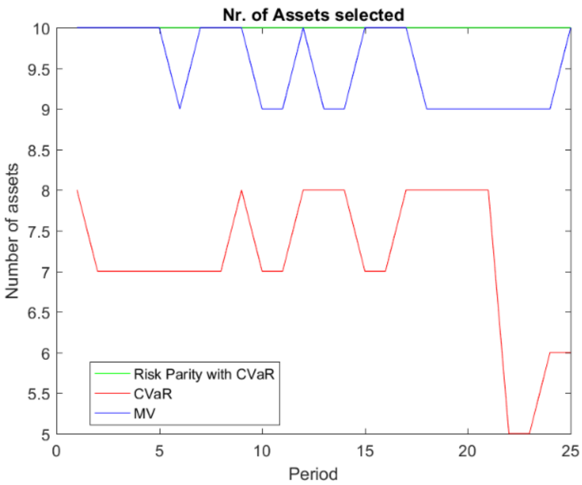
Graph 5. *The turnover of the portolios*



Source: Computed with Matlab

Turnover measures the amount of trading required to rebalance a portfolio to align with a given strategy. Higher turnover generally implies higher transaction costs and more frequent portfolio adjustments. Mean Variance and CVaR are focused in a smaller number of stocks, thus the portfolio turnover will be higher.

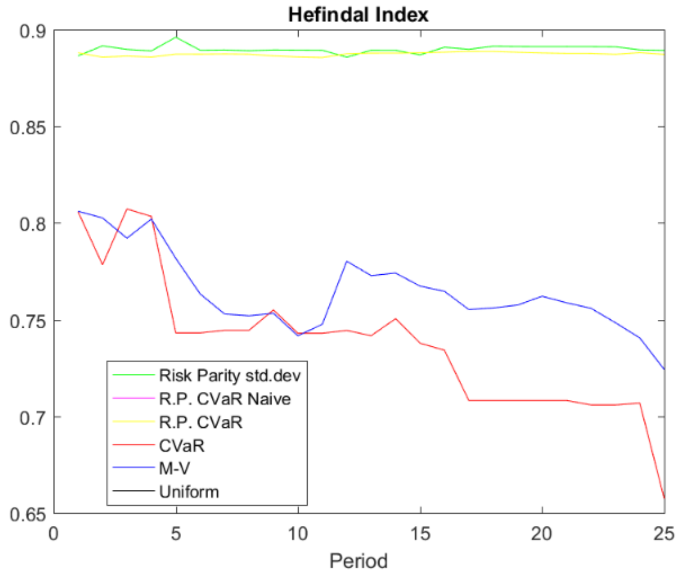
Graph 6. *The number of assets focused*



Source: Computed with Matlab

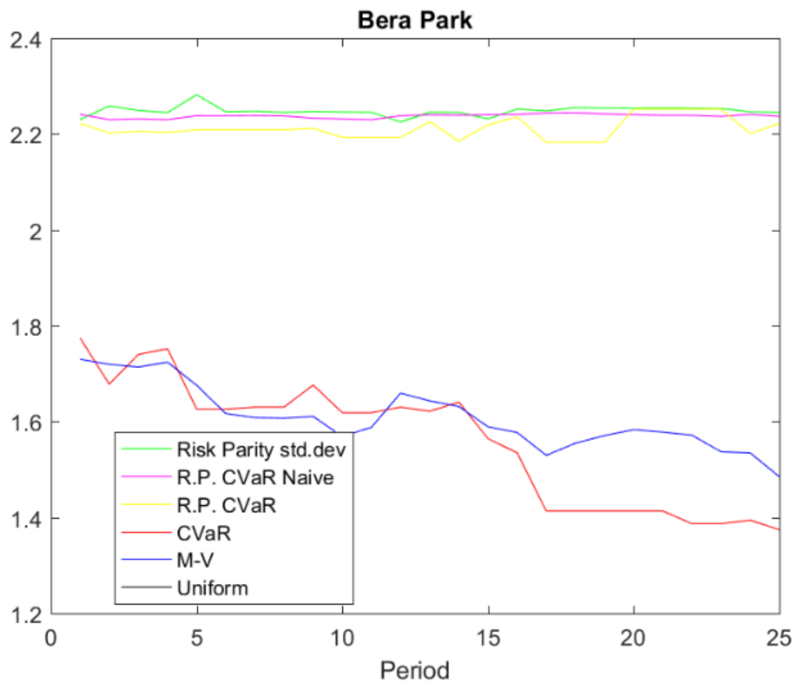
Risk Parity, being less sensitive to extreme market events or return estimates, generally has the lowest turnover. Another way to check the diversification is using the Herfindal and Bera Park indexes.

Graph 7. *The Herfindal Index*



Source: Computed with Matlab

Graph 8. *The Bera Park Index*



Source: Computed with Matlab

The Herfindal (graph 7) and the Bera Park (graph 8) have the same traits with the highest value in the Uniform Portfolio and closer values with Risk Parity strategies. This means that these portfolios are well diversified. The CVaR and mean variance focus on less assets that have the same traits.

All the algorithms are made coding in Matlab without using AI. Today there are several articles that compare the AI role (Tirana & Bejleri, 2024, p.2).

CONCLUSIONS

This study demonstrates the effectiveness of applying the Risk Parity methodology to optimize ETF portfolios, offering a compelling alternative to traditional portfolio allocation techniques. By focusing on balancing risk contributions rather than capital weights, Risk Parity provides a more diversified and stable investment framework. Using ten assets that comprise the EA Bridgeway Blue Chip ETF (BBLU), the research compared various optimization strategies, including Risk Parity with standard deviation, Risk Parity with CVaR, Minimum Variance, Minimum CVaR, and Naïve allocation.

The results highlight the advantages of Risk Parity strategies in achieving superior diversification and risk-adjusted returns. Specifically, Risk Parity with CVaR proved particularly effective in managing tail risks, showcasing robust performance under different market scenarios. Unlike traditional methods such as Minimum Variance, which tend to concentrate heavily on a few low-risk assets, Risk Parity models distribute risk more evenly across the portfolio, mitigating the impact of extreme market events and ensuring a more stable return profile.

The rolling window analysis confirmed that dynamic reallocation based on recent data enhances the responsiveness of portfolios to changing market conditions. While the Naïve strategy provides the highest diversification, it lacks risk considerations, leading to suboptimal outcomes in volatile markets. In contrast, Risk Parity strategies consistently achieved a balance between risk management and return optimization.

This research underscores the utility of ETFs as cost-effective and accessible instruments for implementing sophisticated portfolio optimization techniques like Risk Parity. By addressing practical constraints such as transaction costs and turnover, the study bridges the gap between theoretical portfolio models and their real-world application. This emphasizes also the dynamic of the markets.

It is important to note that in these strategies, the resilience of risk management is also evident in terms of cost-effectiveness, particularly when faced with transaction costs during portfolio calibration.

Also, the financial resilience in these models applied to ETF funds is important in terms of downturns of the markets to mitigate the number of losses of values.

Future research could explore extending this framework to broader datasets, including alternative asset classes and global markets. Additionally, integrating machine learning techniques to forecast risk and enhance allocation models could further improve portfolio performance. Overall, Risk Parity offers a strong foundation for managing ETF portfolios in both institutional and individual investing contexts, providing a pathway for more resilient and diversified investments.

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