Implementation of Leader-Follower Formation Control of a Team of Nonholonomic Mobile Robots

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Abstract: A control method for a team of multiple mobile robots performing leader-follower formation by implementing computing, communication, and control technology is considered. The strategy expands the role of global coordinator system and controllers of multiple robots system. The global coordinator system creates no-collision trajectories of the virtual leader which is the virtual leader for all vehicles, sub-virtual leaders which are the virtual leader for pertinent followers, and virtual followers. The global coordinator system also implements role assignment algorithm to allocate the role of mobile robots in the formation. The controllers of the individual mobile robots have a task to track the assigned trajectories and also to avoid collision among the mobile robots using the artificial potential field algorithm. The proposed method is tested by experiments of three mobile robots performing leader-follower formation with the shape of a triangle. The experimental results show the robustness of formation of mobile robots even if the leader is manually moved to the arbitrary location, and so that the role of a leader is taken by the nearest mobile robot to the virtual leader.

Keywords: Leader-follower, formation, nonholonomic, trajectory control, collision avoidance, multiple mobile robots.

1 Introduction

Leader-follower formation of a team of mobile agents has received special attention from researchers because of its usefulness to many applications, e.g. military applications, transportation, warehouse automation, etc. In performing a formation of team of mobile robots, multidisciplinary technologies of computing, communication, and control need to be conducted. In particular applications, such as transportation, leader-follower formation of multi agents has bounded regulation. For example, in road transportation and warehouse automation, some vehicles should follow particular track, designed by the global coordinator, and have to avoid collisions among others. In this case, some vehicles, which are assigned to follow the same track, can cooperate performing leader-follower formation. One advantage is that it can avoid congestion due to width limitation of the lane. This paper discusses a strategy for a team of mobile robots to perform leader-follower formation. The mobile robot used in this paper is differential-type mobile robot which is categorized as nonholonomic system. Formation control of nonholonomic system
is also of special interest because of its difficulty in controlling a system with nonholonomic constraints. Several methods in control of nonholonomic system have been shown in [12]- [24].

The problem of multi-vehicle formation control has been studied in [16]- [14], and many others, where the focus is on consensus based formation control. Ren and Cao in [16] classify the formation control problems become formation shaping problems, in which the objective control is to establish formation shape, and formation tracking problem, which is to find a control algorithm so that the agents track the predefined trajectories. In [11] and [14], graph theoretic methods and consensus, cooperation in networked multiagent systems are the focus. In the case of road transportation or warehouse automation, the tracks or lanes have been determined and the tasks have been provided by a fleet system or a global coordinator. This paper focuses on the architecture and control strategy for mobile robots in performing leader-follower formation in which the tracks and tasks are predetermined by the global coordinator.

Oh, Park, and Ahn in [13] studies the categorization of formation control which is focused on sensory systems and topology between agents. It leads to three categories, which are position-based, displacement-based, and distance-based formation controls. In the position-based control, agents control to achieve the predetermined formation based on the measurement of their own position in global coordinate system. In the displacement-based control, the measurement used is displacement of neighboring agents. In the distance-based control, the agents control their distance in performing formation. The position-based control needs more advanced sensing capability and lesser interactions inter-agents comparing to the two other methods. The advancement of position sensors have been shown including global positioning system (GPS) [1], inertial navigation system- (INS) [4], vision- [19], [20], laser-based localization [21], [22], and many more. This paper focuses on the position-based formation control.

Several position-based of multi vehicle controls for have been presented in several literature. Dong and Farrel in [6] and Widyotriatmo, Pamosoaji, and Hong in [23] study the position-based method for nonholonomic agents. Ren and Atkins in [15] proposes the position-based method for formation control double integrator agents. The trajectory tracking problem for agent with unicycle-type kinematic model is proposed in [3]. The position based-method requires the ability of mobile agents to measure their position and orientation in the global coordinate.

A system architecture performing feedback coordination between mobile robots and global coordinate system is usually used in the position-based method formation control [7], [26]. The method consists of two steps, which are: first, the global coordinator generate formation virtual trajectories; second, the mobile robots execute control to track the desired virtual trajectory. In this paper, a leader-follower formation control for a group of mobile robots is proposed. The main contributions of this paper is the extension of functions of the global coordinator and of the controller of mobile robots in performing leader-follower formation. The global coordinator performs the virtual trajectories generation of mobile robots. The generated virtual followers are classified by stages. The scheme creates main virtual leader which is the virtual leader for all agents and sub virtual leaders which are the leaders for the consecutive stage followers. In addition, task of global coordinator also assign roles for mobile agents to fill positions of leader or followers. Using the scheme, the minimum communication link between the robots is between the main leader or the sub-leaders and their pertinent followers. The mobile robots’ controller has task to track the virtual trajectory given by the global coordinator, and also executes collision avoidance algorithm using potential field methods [8]- [25], in such a way that the mobile robots track the virtual trajectories while avoids collision among other mobile robots and obstacles.

The rest of the paper is organized as the following. Section 2 depicts the proposed leader-follower formation strategy which consists of system architecture of leader-follower formation, the formation of virtual leader and followers, the role assignment algorithm, and the trajectory tracking control of the mobile robots which also includes collision avoidance strategy. Section
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3 shows the simulation and experimental results. Section 4 draws conclusion and the future research directions.

2 Leader-follower formation control strategy

2.1 System architecture of leader-follower formation

The leader-follower formation control of mobile robots study purposes to design control that drives multiple mobile robots to achieve certain moving or static formation. The strategy of leader-follower formation control of mobile robots can be described as follows. First, virtual trajectories of virtual leader and followers, as well as the formation parameters are generated from a global coordinator system. Several methods in the global coordinator system are artificial potential field, sample-based motion planning, cell decomposition, or others. In this paper, the virtual trajectories of the virtual leaders and followers form a triangle. The formation parameters are assumed to be predetermined from a motion planning algorithm. Second, the role of leader and follower of mobile robots position of leader and followers is voted based on the shortest distance to the leader-follower trajectories. Finally, the leader and followers employ the designed trajectory tracking control and obstacle avoidance algorithms to keep them following individual trajectories while also to prevent them to collide with each other. The architecture of the leader follower strategy is shown in Fig. 1.

![Figure 1: The architecture of the formation control of a team of n mobile robots](image)

2.2 Formation of virtual leader and followers

In this subsection, we consider mobile robots with one leader and two followers performing triangular formation. Then, we show that the number of followers can be expanded by assigning the virtual followers to become sub-leaders.

Let \((x, y)\) be a global coordinate system. Fig. 2(a) shows the formation of one virtual leader and two virtual followers which are located at the first stage follower. The coordinate of the virtual leader is denoted by \((x_{v,1}, y_{v,1})\) and that of the virtual followers are denoted by \((x_{v,i}, y_{v,i}), i = 2, 3\). The virtual leader and two first stage followers form a triangle formation. The velocity and the direction angle of the virtual leader are \(v_{v,1}\) and \(\theta_{v,1}\), and those of the virtual followers are \(v_{v,i}\) and \(\theta_{v,i}, i = 2, 3\).

The distance between leader and follower are denoted by \(b_{v,i,1}, i = 2, 3\). The angle between the line projecting from the coordinate of one follower of the first stage follower to the leader and the x-axis is represented by \(\gamma_{v,i,1}, i = 2, 3\). The angle between the line projecting from the coordinate of one follower to that of the other follower in the same stage and the line projecting from the coordinate of that follower to the leader is denoted by \(\varphi_{v,i,1}, i = 2, 3\).
Figure 2: (a) The configuration of virtual leader and two virtual followers shaping a triangular formation until the first stage follower; (b) the formation configuration is expanded to the second stage follower.

The individual agents of first stage virtual follower become the virtual leader of agents in the second stage followers. Hence the determination of virtual trajectories configuration can be expanded for the succeeding stages (Fig. 2(b)). For the second stage followers, the virtual trajectory 5 can be developed from virtual trajectory 2 as the second follower or trajectory 3 as its first follower.

The virtual leader trajectory is developed by the following equations:

\[
\dot{x}_{v,1} = v_{x,v,1}, \quad \dot{y}_{v,1} = v_{y,v,1},
\]

\[
\gamma_{v,i:1} = \varphi_{v,i:1} + \theta_{v,1}(t) - \pi/2.
\]

The position of the first stage followers at time \( t \) are calculated as follows:

\[
x_{v,i}(t) = x_{v,i}(t) - b_{v,i:1} \cos(\gamma_{v,i:1}(t)),
\]

\[
y_{v,i}(t) = y_{v,i}(t) - b_{v,i:1} \sin(\gamma_{v,i:1}(t)),
\]

\( i = 2, 3. \)

2.3 Role assignment algorithm

Once the trajectories of virtual leader and followers have been developed, a group of vehicles are assigned to those trajectories. Let us consider one virtual leader and two first-stage-virtual-followers as in the previous section. Three mobile robots are to be assigned to individual virtual trajectories. The configuration is illustrated in Fig. 3. Let the position of the \( j \)-th robot be defined as \((x_{r,j}, y_{r,j}), j = 1, 2, 3\) and that of the \( k \)-th virtual trajectory be \((x_{v,k}, y_{v,k}), k = 1, 2, 3\).
The distance between a robot and a virtual trajectory are denoted by $r_{r:v,j:k}$, which is calculated as follows:

$$r_{r:v,j:k} = \sqrt{(x_{r,j} - x_{v:k})^2 + (y_{r,j} - y_{v:k})^2}.$$  \hspace{1cm} (6)

Each robot knows the information of each virtual trajectory position and that of each robot position. The role assignment algorithm is described as follows.

**Algorithm: Role Assignment of Robots-Virtual Trajectories**

1. Calculate the distance for every pair robots-virtual trajectories;

2. Compare the distance for each pair robot to the virtual leader. The robot with minimum distance to the virtual leader is assigned to become the leader;

3. Compare the distance of other robots, which have not been assigned as the virtual leader, to the virtual followers. The robot that has the shortest distance to the first follower is assigned as that virtual follower trajectory. If the distance between the robots are the same, the vehicle with lower assigned number is delegated to that virtual follower trajectory.

The algorithm is checked every loop time so that the roles of individual robots can be changed according to the distance of one mobile robot to a configuration of virtual trajectories.

### 2.4 Trajectory tracking control and collision avoidance of the differential-wheel-type mobile robots

After the virtual trajectories are developed and the robots have been assigned to individual virtual trajectories, the control of a mobile robot to track a particular trajectory is designed. For the first step of control design, the model of differential-wheeled mobile robot is derived. Fig. 4 shows the configuration of a mobile robot.
The coordinate of the mobile robot is \((x_{mr}, y_{mr})\) and the angle of the robot with respect to \(x\)-axis is \(\theta_{mr}\). Let the distance between the left and right wheels be \(2L\) and the radius of left and right wheels be \(R\). Linear and rotational velocities, \(v_{mr}\) and \(\omega_{mr}\), of the mobile robot are obtained by arranging the right wheel velocity \(\omega_R\) and left wheel velocities \(\omega_L\), as follow:

\[
\begin{bmatrix}
v_{mr} \\
\omega_{mr}
\end{bmatrix} = R \begin{bmatrix} 1 & 1 \\ \frac{1}{L} & -\frac{1}{L}
\end{bmatrix} \begin{bmatrix} \omega_R \\ \omega_L
\end{bmatrix}.
\]

(7)

If the linear and the rotational velocities, \(v_{mr}\) and \(\omega_{mr}\), are designed, the solution of the right and left wheel velocities, \(\omega_R\) and \(\omega_L\), are directly obtained as:

\[
\begin{bmatrix}
\omega_R \\
\omega_L
\end{bmatrix} = \frac{1}{2R} \begin{bmatrix} 1 & L \\ 1 & -L
\end{bmatrix} \begin{bmatrix} v_{mr} \\ \omega_{mr}
\end{bmatrix}.
\]

(8)

The mobile robot kinematic equation is derived as:

\[
\begin{bmatrix}
\dot{x}_{mr} \\
\dot{y}_{mr} \\
\dot{\theta}_{mr}
\end{bmatrix} = \begin{bmatrix} \cos \theta_{mr} & 0 & 0 \\ \sin \theta_{mr} & 0 & 0 \\ 0 & 0 & 1
\end{bmatrix} \begin{bmatrix} v_{mr} \\ \omega_{mr}
\end{bmatrix}.
\]

(9)

A reference point \((x_r, y_r)\) is chosen as in Fig. 5, satisfies the following:

\[
x_r = x_{mr} + l \cos \theta_{mr},
\]

(10)

\[
y_r = y_{mr} + l \sin \theta_{mr}.
\]

(11)

The motion of reference point is obtained as follows:

\[
\dot{x}_r = \dot{x}_{mr} - \dot{\theta}_{mr} l \sin \theta_{mr},
\]

(12)

\[
\dot{y}_r = \dot{x}_{mr} + \dot{\theta}_{mr} l \sin \theta_{mr}.
\]

(13)

The linear and angular velocities \((v_r, \omega_r)\) of the reference point are the same as those of
The chosen reference point \((x_r, y_r)\). The relationship between linear and angular velocities and the motion of reference point is obtained as follows:

\[
\begin{bmatrix}
v_r \\
\omega_r
\end{bmatrix} = 
\begin{bmatrix}
\cos \theta_r & \sin \theta_r \\
-\sin \theta_r & \cos \theta_r
\end{bmatrix}
\begin{bmatrix}
\dot{x}_r \\
\dot{y}_r
\end{bmatrix}
\]

(16)

Note that \(l\) can be chosen as a non-zero positive value, thus (16) always has a solution. From (16), if the velocity of the reference point \((\dot{x}_r, \dot{y}_r)\) is designed, the linear and rotation velocities \((v_r \text{ and } \omega_r)\) of the mobile robot can be directly calculated, and so do the rotational velocities of right and left wheels \((\omega_R \text{ and } \omega_L)\) by applying (8). The problem becomes how to obtain a control algorithm so that the reference point of a particular robot follows the virtual trajectory which has been assigned to that robot.

The errors between the reference point of \(j\)-th robot and its assigned \(k\)-th virtual trajectory \((e_{x,j,k}, e_{y,j,k})\) are as follow:

\[
e_{x,j,k} = x_{v,k} - x_{r,j},
\]

(17)

\[
e_{y,j,k} = y_{v,k} - y_{r,j}.
\]

(18)

The differentiation of the errors in (17) and (18) with respect to time can be derived as follows:

\[
\dot{e}_{x,j,k} = \dot{x}_{v,k} - \dot{x}_{r,j},
\]

(19)

\[
\dot{e}_{y,j,k} = \dot{y}_{v,k} - \dot{y}_{r,j}.
\]

(20)
The velocity of reference point \((\dot{x}_{r,j}, \dot{y}_{r,j})\) is designed as
\[
\dot{x}_{r,j} = \dot{x}_{v,k} + K_{P_{x,j}} \left( (x_{v,k} - x_{r,j}) + \frac{1}{T_{I_{x,j}}} \int_{0}^{t} (x_{v,k} - x_{r,j}) \, dt + T_{D_{x,j}} \frac{d(x_{v,k} - x_{r,j})}{dt} \right),
\]
(21)
\[
\dot{y}_{r,j} = \dot{y}_{v,k} + K_{P_{y,j}} \left( (y_{v,k} - y_{r,j}) + \frac{1}{T_{I_{y,j}}} \int_{0}^{t} (y_{v,k} - y_{r,j}) \, dt + T_{D_{y,j}} \frac{d(y_{v,k} - y_{r,j})}{dt} \right),
\]
(22)
where \(K_{P_{x,j}}, T_{I_{x,j}}, T_{D_{x,j}}, K_{P_{y,j}}, T_{I_{y,j}},\) and \(T_{D_{y,j}}\) are positive constants. The control laws (21) and (22) utilize the measurement of robot coordinate \((x_{r,j}, y_{r,j})\) and its correspond virtual trajectory \((x_{v,k}, y_{v,k})\).

Substituting (21) and (22) into (19) and (20), it is obtained
\[
K_{P_{x,j}} \left( e_{x,j} + \frac{1}{T_{I_{x,j}}} \int_{0}^{t} e_{x,j} \, dt + (1 + T_{D_{x,j}}) \dot{e}_{x,j} \right) = 0,
\]
(23)
\[
K_{P_{y,j}} \left( e_{y,j} + \frac{1}{T_{I_{y,j}}} \int_{0}^{t} e_{y,j} \, dt + (1 + T_{D_{y,j}}) \dot{e}_{y,j} \right) = 0.
\]
(24)
From (23) and (24), it can be seen the uniform asymptotic stability of errors \((e_{x,j,k}, e_{y,j,k}) = (0, 0)\) since the values of \(K_{P_{x,j}}, T_{I_{x,j}}, T_{D_{x,j}}, K_{P_{y,j}}, T_{I_{y,j}}, T_{D_{y,j}}\) are positive and thus the equations (23) and (24) satisfy the Routh-Hurwitz stability criterion. Therefore, the errors \((e_{x,j,k}(t), e_{y,j,k}(t))\) converge to zero as \(t \to \infty\) for any initial conditions \((e_{x,j,k}(0), e_{y,j,k}(0))\). The spectrum response of the errors \((e_{x,j,k}(t), e_{y,j,k}(t))\) can also be designed by choosing the parameters \(T_{I_{x,j}}, T_{D_{x,j}}, T_{I_{y,j}},\) and \(T_{D_{y,j}}\).

When the trajectory control is executed, it is possible that the mobile robots collide with each other. To avoid the circumstances, the control algorithms (23) and (24) are modified by applying additional input to repel a mobile robot towards collision. This technique is known as artificial potential field method [17]- [25].

3 Simulation and experimental results

3.1 Simulation of Three Mobile Robots Performing Leader-Follower Formation

The simulation is intended to assure that the chosen of reference point as well as of the control parameters are suitable for the mobile robots. The constant \(l\) which determines the reference point of the mobile robot can be analyzed as follows: the smaller value of \(l\), the motion of robot becomes more oscillation; whereas the larger value of \(l\), the lag between the robot and the virtual trajectory is larger. The motion also depends on the distance between right and left wheels.

From the simulations, the value of \(l\) between 2 cm and 8 cm shows good performance. The controller parameters \(K_{P_{x,j}}, T_{I_{x,j}}, T_{D_{x,j}}, K_{P_{y,j}}, T_{I_{y,j}},\) and \(T_{D_{y,j}}\), \(j = 1, 2, 3\) are designed by using the pole placement method with the desired poles of equations (23) and (24) are: \(-0.2\) and \(-0.01\). Thus, the control parameters are obtained as follow: \(K_{P_{x,j}} = K_{P_{y,j}} = 0.21, T_{I_{x,j}} = \ldots\)
$T_{Ig,j} = 105.0$, and $T_{Dx,j} = T_{Dy,j} = 0.01$ for $j = 1, 2, 3$. Using these parameters, the simulation is shown in Fig. 6. For the assigned three virtual trajectories $v_1$, $v_2$, and $v_3$, the robots $r_1$, $r_2$, and $r_3$ which are started from different locations successfully perform leader-follower formation. The control inputs given to right and left wheels of individual robots stay below 14% of their maximum power which is suitable for the design (Fig. 7).

![Figure 6: Robots r1, r2, r3 performs leader-follower formation using the proposed method.](image)

![Figure 7: Motor power given to individual motors (left and right wheels) of robots r1, r2, r3 in performing leader-follower formation of Fig. 6.](image)

3.2 Experiments of leader-follower formation using three mobile robots

Fig. 8(a) shows the experimental setup which includes the arena, three mobile robots, camera-based localization system, and personal computer (PC) as the global coordinator system. The
arena is colored white, and three robots are colored with three different colors: red, green, and blue. The localization sensor is a camera mounted at the height of 114 cm. The camera is utilized to differentiate the colors of individual mobile robots and of the arena. Two colors patched in a mobile robot are utilized to measure its orientation. Fig. 8(b) shows the picture from the camera point of view. The position and orientation of all robots are calculated in a microcontroller from the measured position of colored rectangular shape patched on the mobile robots. Then, the control input for each mobile robots are is calculated and sent to mobile robots using wireless Bluetooth communication. The control parameters used in the experiment are the same as those used in the simulation.

![Experimental setup](image)

(a) The experimental setup consists of arena (white pad), three robots, camera, and PC as the global coordinator system; (b) Top-view picture is captured from the camera. The position of front and back rectangular is obtained so that the position and orientation of the three mobile robots can be calculated.

Fig. 9 shows the first experiment of three mobile robots performing leader-follower formation without being disturbed. The mobile robots are initially located near the virtual trajectories. From Fig. 9, the mobile robots perform leader-follower formation following the given individual virtual trajectories. Using the proposed control method, the mobile robots r1, r2, and r3 follow the pertinent virtual trajectories v1, v2, and v3, respectively. In this case, the mobile robot r1 becomes the leader and mobile robots r2 and r3 become the followers, all together perform triangular formation. Fig. 10 shows the control signals, which are power signals, given to the motors of mobile robots for this experiment.

In the second experiments (Fig. 11), while three mobile robots perform leader-follower formation, the leader, mobile robot r1, is intentionally disturbed by moving its location the back. In this case, the virtual trajectories are still moving forward. The role assignment algorithm is performed so that the r2 becomes the leader and the r1 replaces the position of r2 in the formation. While tracking the new trajectories, the mobile robot r1 also performs obstacle avoidance algorithm so there is no collision between r1 and r2. Fig. 12 shows the control signals which are the power given to the individual motors of mobile robots for this experiment.
Figure 9: The motion of three mobile robots in the first experiment: Three mobile robots are located in various location and then performs leader-follower formation.

Figure 10: The motion of three mobile robots in the first experiment: Three mobile robots are located in various location and then performs leader-follower formation of Fig. 9.
Figure 11: The motion of mobile robots in the second experiment: the leader r1 is intentionally disturbed by moving its location to the back. The role assignment algorithm is performed in which r2 replaces the leader formation and r1 replaces the r2 position in the formation.

Figure 12: The power signals given to the individual motors (left and right wheels) of the mobile robots in performing leader-follower formation of Fig. 11.
4 Conclusions and future work

In this paper, a strategy of leader-follower formation of mobile robots is presented. A role assignment algorithm is proposed to allocate mobile robots following individual virtual trajectories in the leader-follower formation. A trajectory tracking control law is developed. Control parameters are designed by the pole-placement method for mobile robots to track their individual virtual trajectories. The combination of trajectory tracking control and artificial potential field assures that the mobile robots track the pertinent virtual trajectories and avoided collisions among others. The proposed method is tested by simulation and experiments of mobile robots performing leader-follower formation. The method is also tested by experiment in which the leader is intentionally disturbed by moving the leader to the back. By the implementation of the role assignment algorithm, the mobile robots successfully switch their position to maintain the leader-follower formation.

The future works include the integration of the motion planning and control which can handle double collision avoidance layers. One collision avoidance layer lies on the motion planning which is designed at the trajectories generation, and the other is on the low level controller as it is implemented in this paper. By establishing the double collision avoidance layers, the safety is more guaranteed and thus the implementation of multiple wheeled vehicles performing leader-follower formation is closer to a real application.

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